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A Study of the Use of Teacher Questioning
to Promote Reasonable Mathematical Answers from Sixth Grade Students

Abstract

In this action research study of my sixth grade mathematics class, I investigated the influence a change in my questioning tactics would have on students' ability to determine answer reasonability to mathematics problems. During the course of my research, students were asked to explain their problem solving and solutions. Students, amongst themselves, discussed solutions given by their peers and the reasonability of those solutions. They also completed daily questionnaires that inquired about my questioning practices, and 10 students were randomly chosen to be interviewed regarding their problem solving strategies. I discovered that by placing more emphasis on the process rather than the product, students became used to questioning problem solving strategies and explaining their reasoning. I plan to maintain this practice in the future while incorporating more visual and textual explanations to support verbal explanations.

Asking oneself “is this reasonable” happens many times throughout a given day. Is what I am wearing reasonable for today’s weather? Did I workout for a reasonable amount of time in order to reach my weight-loss goal? Did I pack a reasonable amount for lunch to hold me over until dinner? Or did I complete a reasonable amount of homework problems to convince the teacher that I tried but did not understand all of it? Questioning reasonability happens naturally; yet, somewhere along the line students have been allowed to develop a habit of not questioning the reasonableness of solutions of mathematics problems. Two questions: *Did I get the right answer*? or *Did I do it right*? have become common replacements. However “is this reasonable” and “is this right” are not synonymous. The former requires the problem solver to fully understand what is being asked of them, actively think about the steps they took, assign value to the numbers and calculations, and make sure that the task and steps justify the result. The latter requires confirmation from an outside source relieving an amount of responsibility and ownership.

The National Council of Teachers of Mathematics (NCTM) principles and standards challenge educators to help students learn with understanding, recognize reasoning as a fundamental aspect of mathematics, and evaluate mathematical arguments (NCTM, 2000). I have failed my students if I continue to let “is this right” become the norm while “is this reasonable” fade into nonexistence. In my undergraduate years while interacting with young students during my practicum and student teaching field experiences, one thing my supervisors and cooperating teachers praised me on was how well I questioned students. I frequently asked “why” of my students, just like I was a 2 year old beginning to question the world and the way things were. Now that I have my own classroom and the burdens that come with being a teacher, I have allowed pressures, constraints, and in all honesty, laziness, let “yes,” “no,” and “thanks for trying" replace "why" in my instructional vocabulary. The absence of requiring students to address "why" has resulted in many of my students performing meaningless calculations. The outcomes hold no importance other than just being the answer they got. The following is an excerpt from one my personal journals that demonstrates the problem.

> Because I did not emphasize double-checking variable solutions and expressions students were doing work that initially appeared to be correct but actually did not make sense when applied. For example when given a table such as this, they might say the missing expression is $w - 20$ because $30 - 10 = 20$ but when asked if this works for the next w value, a response I received was "I don't know, I didn't try it." When given the equation $32/n = 4$ some would say $n = 128$ because $32 * 4 = 128$. They were sticking to performing the inverse operation and did not plug their value back in for the variable to make sure it worked. To the students that I was able to catch doing this I simply said so you are telling me that $32/128 = 4$, does that make sense? They realized that it did not, some briefly protested saying "but I multiplied" and then they said "I should divide by 4, huh." (Personal Journal, 09/29/08)

I acknowledge that initial questioning must begin with me; however, I want to use research to help me find a way to get more of my students to that point of asking "is my answer reasonable?" I want students to think before they act. I want my students to internally ask, *what is the problem asking me to do, what am I going to do to solve it*, *why am I choosing this method,* and *does my answer make logical sense?* Through my action research, I want to learn what type of instruction, classroom environment, and student tasks will best support this type of self-questioning. To begin, I planned to examine what would happen to my students' ability to check the reasonableness of explanations and problem solving methods when I changed my questioning techniques. Succinctly, I desired to decrease my students' dependence on me, or some other third party, and help them become justifiably self-reliant on their own mathematical reasoning.

My research took place in my sixth grade classroom of 25 students. Approximately 900 students attended the sixth through eighth grade Title 1 building, located in a large city in Nebraska, during the 2008-2009 school year. The students were of varying ability levels assigned to a general sixth grade mathematics classroom. None of the students received differentiated mathematics instruction; however, a few received extra support in the supplementary course titled Math Intervention. Teachers used the intervention class to reinforce basic mathematical skill and content retention. Nearly half of the students in my class were of European descent while the other half was a mixture of Latin, Asian, and African backgrounds. At least three were currently involved in the English Language Learners program; however, there were more students whose primary language was different than English. Languages spoken in addition to English were Bosnian, Spanish, Arabic, Kurdish, Karen, and Nepalese.

Problem Statement

Having students ask and answer questions related to the reasonableness of a solution or strategy is applicable to literally all walks of life, not just to the field of mathematics. Educators desire to help students become autonomous adults who are fully capable of determining if they have put forth an ample amount of and accurate effort into a given task. Furthermore, it is frustrating and overwhelming for teachers to do their own thinking and the thinking of 30 students daily throughout the school year. I want students who can reason through their own actions before determining that they need outside input. Facilitating students in advancing their reasoning skills also helps make the NCTM standard of communication come alive. You cannot coherently communicate your mathematical thinking if no actual thinking took place. For decades, our educational system has been able to teach the majority of students “how” to perform operations and memorize general situations when those operations should be applied. Drill and practice has held a long-standing position in the history of education. Today, however, these old tactics are no longer sufficient. We are requiring more of students by way of complexity, conceptual knowledge, and how soon they are introduced to topics such as algebra. Yes, we still want students to know how to perform the basic operations, but more importantly we want them to become problem solvers. Before students can become efficient problem solvers they must first be able to reason, to ask what is the problem, what approaches can be taken to solve it, and what kind of results will be deemed acceptable.

Literature Review

As a mathematics educator, I accept the challenge given by the NCTM principles and standards to have students learn with understanding, recognize reasoning as a fundamental aspect of mathematics, and evaluate mathematical arguments (NCTM, 2000). In order to meet this challenge I realized that I first needed to be reminded of and relearn how to question students and how to respond to their comments and inquiries. In reviewing literature to better understand how I could change my questioning habits to help lead my students toward being autonomously able to determine the reasonability of a solution, three main themes were persistently addressed: the role of the teacher, the role of and expectations placed on students, and content and instruction that are aligned with NCTM guidelines.

The Role of the Teacher

The role of the teacher in the classroom has shifted in the last decade or two. Gone are the days of standing in the front of the classroom instructing on what to do while students feverously take notes that will later be followed by drill and practice. According to Peterson, Fennema, Carpenter, and Loef (1989), "[T]he teacher's role is one of facilitating the construction of student understanding and knowledge" (p. 37). Through questionnaires and interviews, Peterson et al. (1989) studied first grade teachers to understand what effect teachers' pedagogical content beliefs had on their decision making, thinking, teaching, and students' learning and achievement in regards to addition and subtraction. They found that the beliefs of more experienced teachers where closer to a constructivist perspective than that of less experienced teachers. This led them to believe that "teachers' pedagogical content beliefs and their pedagogical content knowledge seem to be interrelated" (p. 38). Fraivillig, Murphy, and Fuson (1999) conducted a case study of one expert teacher's methods to see how to "effectively advance children's mathematical thinking in inquiry-based mathematics classrooms without undermining children's intellectual autonomy" (p. 149). In agreement with the authors of the study on teachers' pedagogical content beliefs, Fraivillig et al. (1999) emphasized that not only does the teacher need to be a facilitator of discourse but she also needs to establish and guide development of social norms and support students' understanding. When this is done, three components will be apparent in the teacher's practices: eliciting student's solution methods, supporting student's conceptual understanding, and extending student's mathematical thinking.

Two things must take place as teachers switch gears from telling students what to do to helping students construct their own knowledge by using what they already know to successfully navigate the waters of the unknown. Teachers must first become better questioners, listeners and responders. Second, they must use explicit strategy instruction and have it become a common practice according to Nicol (1999). Nicol (1999) reports on a curriculum and instruction course that she co-designed and co-taught for prospective teachers. In her report, she discusses the difficulties prospective teachers faced in their efforts to have students actively participate in mathematical thinking and dialogue. Nicol observed that questioning serves one of three main purposes: to learn what students are thinking, to get students to the answer, or to test students thinking. After posing the initial question, the teacher has to have a deep understanding of the mathematics to fully listen and respond to what the student's answer is and where that answer will take the discussion.

In helping students construct their own understanding and use their prior knowledge to do so, the teacher must provide students with explicit strategies that they can employ to be successful. Knowing what needs to be done and how to carry it out in solving a mathematical problem are not innate (Goldman, 1989; Pape et al., 2003). Goldman (1989) examined strategy instruction research in mathematics, more specifically the implications this research held for learning-disabled students. She concluded that "procedures that merely instruct the learner in *what* to do are inadequate; instruction in *how* to do these things is necessary" (p. 53). Pape, Bell, and Yetkin (2003) further stress this point by commenting: "For some students, this lack of explicitness may hinder their ability to reach their full potential" (p. 180). Pape and Bell constructed and then implemented a teaching experiment during a two-year professional development program in which they were both participating. They sought to create a learning environment that produced self-regulated learners in Bell's pre-Algebra and regular seventh grade mathematics classrooms with the use of explicit strategy instruction and student record keeping of the strategies that they used. Good, Slavings, Harel, and Emerson (1987) came to a similar consensus in their study of student passivity. They looked across age, ability level, and gender to determine which students where asking questions and what kinds of questions were being asked. The researchers were discouraged by how infrequently academic questions were being asked and suggested that teachers teach students how to ask questions.

As teachers are working on becoming facilitators of student learning and classroom discourse, instructors of strategic problem solving steps and behaviors, and expert questioners, it is essential that they also establish classroom norms that create a supportive learning environment. Pape et al. (2003) would agree that it is on the shoulders of the teacher to scaffold and create learning environments that support student participation and mutual respect between all involved parties. With students receiving explicit instruction on how to be successful, teachers can then raise their expectations for all students with confidence, knowing that students are equipped to reach those expectations

Expectations of Students

While the role of the teacher develops from instructor into facilitator and supporter, the role of students is also changing. Students need to move beyond being passive learners to active learners. Passive students do not volunteer or respond when called on, ask few questions, and approach the teacher infrequently (Good, et al., 1987). An active learner is one who will "analyze mathematical situations, critically examine their mathematical thinking and that of their classmates, and explain and justify their mathematical reasoning" (Pape, et al., 2003, p. 183). Peterson et al. concur in saying that "…the student's role is one of engagement in active cognitive learning..." (1989, p. 37). Students must expect to be actively involved in the mathematics that is taking place in the classroom, not merely regurgitate information, observe, and occasionally record.

One can naturally infer that if teachers are raising their expectations of students' capabilities then students should produce more. Producing more does not mean more paper-pencil work but instead that students should be engaged, explaining and justifying problem-solving methods, making sense of peers' methods, working collaboratively, and challenging the solutions and methods of peers (Fraivillig et al., 1999). Krebs (2005) reported on the experiences of 20 middle grade teachers as they studied the performance of pairs of students working on a challenging mathematical task. In her study, it was readily apparent that students needed to keep complete records of their thinking so that their peers and teachers might fully understand their mathematical processes and reasons. Fuchs et al. (1996) studied peer-tutoring interactions to "examine the quality and effectiveness of students' mathematical explanations as a function of student ability" (p. 634). They noticed that the student who constructs the explanation achieves greater understanding than the listener.

As students grow in their ability to fully communicate their mathematical thinking and practice examining the thinking of their peers, they mature into what Goldman (1989) refers to as "good strategy users" or what Pape et al. (2003) call "self-regulated learners". These are students that have a variety of procedures at their disposal, are flexible with those procedures, actively monitor if the steps they are taking are getting them to their desired end, and understand that academic learning is a proactive activity that requires inner motivation and strategic behavior.

Mathematics Content and Instruction Aligned with NCTM Standards

Intuitively, what is taught and how it is taught cannot remain stagnate if the role of the teacher and students is shifting. Math is no longer merely seen as facts and procedures, it includes "learning to reason statistically, to think algebraically, to visualize, to solve problems, and to pose problems" (Pape et al., 2003, p. 180). Fuchs et al. (1996) effectively describe what mathematics should be in the classrooms of today:

> The central assumptions underlying this series are that solving problems related to everyday life should be the primary focus of mathematics instruction; reasoning about mathematics, rather than memorizing rules and procedures, helps children

> make sense of mathematics; mathematics is a way of thinking and a network of related ideas and concepts, as well as a vehicle for developing critical thinking, creative thinking, and decision-making abilities; manipulatives are a powerful tool to help children link concrete objects to pictorial representations and finally to abstract symbols; and computational proficiency is a necessary tool for successful problem solving. (p. 638-639)

Challenging and meaningful mathematics that elicit discourse on problem solving strategies and encourage multiple approaches should be found in every mathematics class. These kinds of activities allow all students at any ability level to get involved, be challenged to go deeper into the mathematics, and increase their understanding of the mathematics. Teachers in Krebs' (2005) study found that much insight could be gained from even the partial or incorrect solutions of students. To help all students enter into these complex task Goldman's (1989) summative report reminds us of the problem solving steps of Polya (1957): understand, plan, carry out and verify, and those of Graofalo and Lester (1985): orientation, execution and verification. These frameworks give students a procedural attack plan when approached with the kind of mathematics NCTM standards propose (NCTM, 2000). Placing vigorous mathematics at the core of our curriculum will aid in reaching all learners, drawing the most out of learners, and supplying a base for teachers to extrapolate from.

Purpose Statement

The purpose of my study was to look at sixth grade students' ability to determine if an answer is reasonable after a change in teacher questioning had been implemented. The literature emphasized the importance of a challenging mathematics content, requiring students to explain their thinking, establishing classroom norms that encourage discourse and participation, giving explicit instruction on strategic behaviors, and supporting students as they construct their knowledge. Fuchs et al. (1996) learned that the quality of student explanations play a key role in the understanding of the listener. My interest in students' explanations was to see if their reasoning helped them determine if their results were logical. Nicol (1999) stressed the importance of teacher questioning matching the intended purpose of the questioning. Fraivillig et al. (1999) state that effective teachers are able to elicit solution methods, facilitate student responses and support students' understanding. I recorded my actions and questioning tactics to see how it impacted students' ability to conclude if an answer was reasonable. The teaching experiment of Pape et al. (2003) was closely related to my interest as they sought to grow their students into self-regulated learners. However, the main goal of that study was to have students become aware that their actions, or inactions, had a direct effect on their academic outcomes by having students keep track of the problem solving strategies they used. I wanted to see if students' actions gave them confidence in the soundness of their problem solving steps and solutions. To accomplish this, I examined students' abilities to question problem solving approaches and results as well as to explain their own problem solving methods. I also examined my questioning tactics as the teacher. I attempted to answer three research questions:

- What will happen to students' reasoning and questioning (of themselves and others) related to problem solving after a change in teacher questioning has been implemented?
- What will happen to student's explanations of their problem solving methods when asked to justify or elaborate on their results?
- What happens to my mathematics teaching when I implement probing questioning tactics in response to student's problem solving and solutions?

I desired insight as to how to decrease students' dependence on an outside party to validate their problem solving and help students become justifiably self-reliant on their own mathematical thinking.

Method

To help answer my research questions, I collected data from a variety of instruments from late February 2009 through mid-April 2009. The instruments consisted of my daily notes, weekly teacher journals, student interviews, daily student questionnaires, and end of chapter test questionnaires. Data was supported with work done by students during the warm up/exploration activity, daily journaling/note taking, homework checking, and Friday journaling.

My daily notes generally consisted of the daily topic and intriguing questions or problem solving methods offered by students. The daily notes were very brief and served the purpose of helping me write a more formal journal entry at the end of the week. In my weekly journals (see Appendix A for weekly journal prompts), I discussed the general mathematics concepts focused on for the week, memorable student questions and comments, noticeable changes in students or myself in regards to my research focus, conflicts of being both teacher and researcher, perceived limitations of the unanalyzed data I had gathered so far, and possible ways to improve upon those limitations in the upcoming week. I supplemented the content of my journals with the work done by students as mentioned above. The work, with the exception of the warm up/exploration activities, was primarily done on marker boards; thus, photographs were taken to preserve them for later analysis.

Friday journaling was a form of student work that was not carried out as often as planned due to chapter testing frequently arriving at the end of the week. A strong effort to rectify this was not made since students were not giving thoughtful written journal responses. Toward the end of the data-collecting period an attempt to change the instrument was made to elicit verbal descriptions of student reasoning and to increase participation.

After the first chapter test questionnaire, I took class time to discuss with students their perception of the questionnaire and to ask for any recommendations (see Appendix B for first test questionnaire). A major complaint was that it was difficult to recall information regarding my questioning habits at the end of the chapter. Students wanted to be asked on a daily basis instead, while it was still fresh in their minds. Therefore, I created a daily questionnaire that the students were supposed to answer and the end of each day regarding my questioning habits and student explanations (see Appendices C-E). The daily questionnaire was printed on the back of each day's warm up/exploration activity that would lead into the day's lesson. Upon entering the data from daily questionnaires into spreadsheets it became apparent that the majority of students did not complete the questionnaires or put very little effort in to doing so. The test questionnaire gave a broader look into student reasoning and their perceptions of peers (see Appendices F-G). My questioning tactics, classroom practices and philosophy were also addressed on the test questionnaires. Students gave more in-depth answers on these, which may have been influenced by the fact that they were attached to their chapter test. Both questionnaires were anonymous and helped fill the void of a daily log of questioning interactions that I had planned but was unable to implement.

My initial goal was to keep a daily log of the kinds of questions asked and who asked them. A template was made to assist me in carrying out this goal (see Appendix I). During the first day of implementation it became immediately apparent that this record keeping would not be feasible. The difficulty of instructing, assisting, supporting and responding to students while simultaneously attempting to record every questioning interaction was too great. The daily and test questionnaires allowed for the same kind of information to be gathered by students, yet unfortunately, with less accuracy and increased subjectivity.

Before beginning my research, I was granted IRB approval for my study and could use data and information given by students who, along with parents, gave consent. All data collection instruments pertaining to the students were anonymous with the exception of 10 voice-recorded student interviews that were conducted; therefore, pseudonyms are used throughout this paper. The 10 students interviewed were randomly chosen by another teacher in the building who had distributed and collected consent forms from students. The 10 students were a sub-group of the total amount of students whose parents had given consent for participation in my research. The initial plan was to ask students a specific list of questions regarding their problem solving and reasoning on objective test problems (see Appendix A). However, the fourth student I interviewed suggested that I use the topic of their most recent journal entry, integer operations, for the interview topic. Therefore, the majority of the interviews (seven of the 10) were less structured and focused on student's reasoning over one of the four main operations (addition, subtraction, multiplication, or division) as applied to integers. Each interview was about 10 to 15 minutes long, with the exception of two that ran 24 minutes and 47 minutes each. The lengthier interviews occurred after school while the others had to be done within a 15-minute lunch break. The only record of the 10th interview was the work the student did on paper due to an accidental deleting of the voice recording immediately after the interview ended.

Findings

A typical day during my research study began with students coming into my sixth grade mathematics classroom, picking up their Effort Calendars and the warm up/exploration from the counter, sitting down and getting to work. After the daily announcements were read over the intercom we discussed the warm up/exploration since it led into the topic for the day. A transparency copy of the warm up/exploration they were working on was projected on the overhead and students were chosen to come up and write their answers. I usually chose the first group of students to go up because a variety of factors, including if the student had the work completed, solved the problem in a manner different from their peers, or if they had not done the work and I wanted to get them actively engaged. If there were a sizable quantity of problems to do, students just passed the marker on to someone of the opposite gender who had not gone up yet. Once answers and work, were displayed we would then discuss what was asked on the sheet and determine if answers were correct. The focus was on why the answer was or was not correct and how solutions were found.

The warm up/exploration activity led right into note taking in their math journals for which they copied down the chapter title and new vocabulary. The class then used a related scenario I gave them to create definitions and examples of the vocabulary to record in their journals. For instance the topic for chapter 8.10 was Percent Problems in which students were to use what they learned about percentages to find tips, sales tax, and discounts on bills. In trying to define tip, students brought up that it was additional money left after the bill. After vocabulary transcription, discussion and examples were completed homework was addressed.

If the previous day's homework raised questions that I would like the entire class to be aware of or contained an important concept that I wanted to make sure all students understood, we would briefly go over it before they were turned in. I did not provide homework answers but instead had students come to the board to solve and explain their solutions. Their peers were allowed to ask for clarification or express concerns at that time. Every student, even those who came to class unprepared, was required to take part in the reviewing of the homework. If there was no homework assignment or questions from the students regarding their homework, students completed the daily questionnaire on the back of their warm up/exploration and then got started on their assignment for that evening. The homework was very concise, usually four to six problems, so students quickly grouped together to assist each other in getting it done. Upon leaving the classroom at the end of the period students had to answer a question related to the day's concept to get out the door. The exit question may have been to tell me what method they preferred to solve a particular kind of problem, demonstrate a vocabulary term, or provide an accurate response to a closed question to name a few.

The most common causes for changing the daily routine were Friday journaling, testing, unfinished business, or concepts that needed further exploration. On Fridays, instead of taking notes in their journal students responded to a given open prompt. The warm up on testing days was a review of the material included on the test. There were no daily questionnaires for students to complete; however, the test questionnaire was attached as the last page of the test (see Appendices B–E). If I determined that the previous day's topic needed to be carried into the following day we usually began that next day with completing the unfinished vocabulary or an activity to help students explore the concept even further.

The most significant change from the usual daily classroom routine was the amount of time spent on the warm up/exploration activity. Instead of the daily lesson consisting of an introduction/opener followed by guided practice, independent practice, and then a closing, I focused on students' understanding and explanations of the introductory activity. This gave me insight into the students' prior knowledge and allowed them to discuss the mathematics, question one another, and construct their own meanings. In this manner, the daily verbal mathematics discourse gave me information that would help answer my research questions.

What happened to students' reasoning and questioning?

My first mission was to re-implement "why" back into my instructional vocabulary. I responded to any answer or partial explanation given by a student with "why" or with feigned ignorance. For instance on March 30, one student discovered that when adding integers it does not matter which number she began with so she preferred to use the number in parentheses to coincide with the order of operations. Another student asked if that would work with subtraction and I replied "I don't know, will it?" Since I was not giving students direct answers to their questions they were forced to reason to answer their own question or lean on the input of their peers to build a more complete understanding.

Many of my students quickly internalized my actions and became very outspoken about letting their peers know when an explanation a peer gave did or did not make sense. On a student test questionnaire given February 16 following the first part of the chapter eight test, I asked, "If you could only pick 1-3 peers from class to explain how they solved a problem who would it be? Why?" Derrick[1] responded, "Karen because she can explain very well and makes sense and some other kids make it hard to understand what they're trying to say." Later that same month, two students responded on their daily questionnaire to a question asking who gave a really good explanation in class: "Nooren, wrote it on the board; it was visual" and "Nooren because she made it understandable and she explained the 2 differences." Students were able to identify which of their peers gave useful explanations and even identify characteristics of those explanations that made them easier to comprehend. Not only were students able to pick out whose explanations were helpful they could also discern which ones added to their confusion. For example, in my March 10 teacher journal following an introductory lesson to integers, I wrote, "The class couldn't define (or give words) for opposite so I made it their homework.

[1] All names are pseudonyms.

Devin's definition was that positive and negative numbers were mirror images of each other but his classmates argued that mirrors show the same thing." Peers questioning peers and then responding to those questions became an expected aspect of math class.

Students knew that their solutions and problem solving methods would be scrutinized. I found that my students would not offer their reasoning as an absolute; rather they expected that changes would be made. My daily notes on April 17 provide a snapshot of what this looked like in practice.

> Upon seeing the picture on the board (Appendix J) along with the statement that Lisa had measured the four angles and found their sum to be 310º. Students were asked if they believed Lisa was right and explain how they knew. Together Karen, Lona, and Shayla said Lisa was correct and explained their reasoning to the class. After hearing Angie's reason that together the angles create a full turn which is 360º, the three ladies changed their previous argument to say that Lisa was incorrect because the sum of angles 1 and 2 were 180º and so was the sum of angles 3 and 4. When I asked students to go to different areas of the room that represented the argument that convinced them of the Lisa's accuracy or inaccuracy the three ladies amended their position again when Boyd pointed out that their idea and Angie's was basically the same. They concurred. (Teacher Journal, April 17, 2009)

The example described demonstrates how students were comparing and contrasting peers' explanations in order to synthesize their own understanding and amend previous conclusions. On a test questionnaire given 10 days earlier, students also showed that they were internalizing the belief that initial answers are still a work in progress. The following are student responses to the question “after solving a problem do you ask yourself if your answer makes sense?” on the test questionnaire given April 7:

> “Yes, because sometimes it won’t.”
>
> “Yes, because if it doesn’t then it wrong.”
>
> “Sometime it could be big or small”

Their responses support the assertion that students do not believe their initial answers to be final. In an interview on April 7, Boyd contemplated the answer to two integer subtraction problems.

> “Five minus negative two is three because if I subtract two I get three. But if I subtract negative two I get seven. But I think it is three because it is subtracting.” Earlier he had solved 5 + (-2) so I wrote 5-(-2) =3 and 5+ (-2) =3. In seeing this he said “I’m sticking with this [5+ (-2) =3]…subtracting go to the left but since you have a negative it would just go to the right and you end up at seven.”

He too was able to take the information, rethink his previous work, and come to a new solution that made more sense when presented with the written equations. Students were beginning to understand that problem solving in mathematics was similar to writing a paper in language arts class. They both required outside input, editing, and revising. An initial solution or method was not final but by hearing peers’ comments and questions, students became more able to formulate clear and accurate explanations.

What happened to students’ explanations?

After I changed my questioning tactics, students began to expect to be questioned by their peers, as well as me, upon volunteering an answer to a problem. A constant theme in our classroom centered on *showing your work*. One of the questions on the student test questionnaires was, “which is valued more math class, right answers or explaining/showing what to do to get the answer?" From test questionnaires that were collected from chapters seven, nine, and eleven, 80 out of 92 students marked explaining/showing as *more valued*. Sample student responses in March and April included:

> "Explaining, anybody can know but only aware people can know how."
>
> "Showing work cause Miss Grayer always says, 'show work, show work, show work' never 'get the right answer."
>
> "Explain/showing cause Ms. Grayer always said How or Why or Show you work."
>
> "No one knows how you did it without the work."
>
> "If you say the right answer you've learned just the right answer but if you're wrong but have it explained, you might not only learn the right answer but a different way of getting it as well."

Students were internalizing, or at least understanding, that answers alone were not enough in mathematics. It was the explanations behind the solution that gave the answer validity.

In my conversations with students I could also tell that students were becoming used to offering explanations. On March 13 in my teacher journal I wrote;

> I do think [the students] are getting used to showing work and [my asking] why? I asked Cage a question, he answered. I said why, he answered again. I said why [again] then he said, "we could go at this all day Ms. Grayer." (Teacher Journal, March 13, 2009)

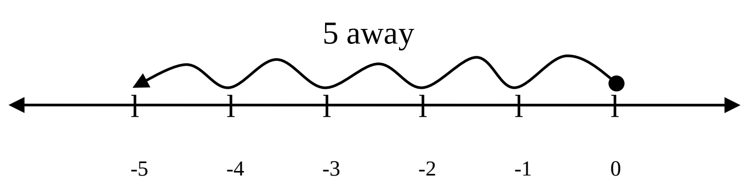
Even though Cage responded with humor by the third round of me asking “why”, being probed for more did not agitate him. His actions alluded to the fact that it was becoming routine for students to respond to questioning from me.

Backing up answers with an explanation became so commonplace in my mathematics classroom that if I walked up to the board, after students had written warm up answers, with a frown on my face, he or she knew something was wrong. For example, Brenda quickly blurted, “there’s no work” before I could even verbalize that something was missing.

My desire for students to explain themselves was even apparent in my instruction. As a Friday journal in March I wrote the following prompt on the board for students to respond to in their journals: *A) The opposite of -5 is 5. B) The absolute value of -5 is 5. What is the meaning of [the answer] 5 in both problems?* The students came up with a journal response as a class as a model of what to do for future journaling.

A) The 5 is the same distance as -5 but on the other side of 0.

B) Absolute value is the distance from 0.



The answer to both questions in the prompt was five but I created the question to emphasize that it is not the answer that has value in itself but the understanding and reasoning behind the answer that gives the answer merit. Students responses on the questionnaires, combined with Cage’s compliance with being probed to verbalize his reasoning and Brenda’s automatic knowing that showing work and solutions go hand in hand, helped point to the idea that students were expecting that answers alone were not "good enough" but that the reasoning behind the answer was what was important.

What happened to my mathematics instruction?

Implementing probing questioning tactics in response to students' problem solving and solutions influenced my mathematics instruction. Students gave more value to the reasoning behind answers because I give more weight toward how students arrived at answers as opposed to the answer they got. On a chapter 11 test questionnaire one student replied that they knew explaining/showing what to do to get the answer was more important in our math class "…because Ms. Grayer says she loves it when she sees work." I let students know by my words that their reasoning was what was of more importance. In my daily notes on March 13 I wrote:

> In class we were going over a homework problem, asking students which given series of integers were ordered from least to greatest that I noticed many of the students had missed. We began with the choice A and students told me why it was incorrect. On choice B [Angie] said it was the right one. When I asked her why she knew the answer was B she said, "Because you didn't mark it wrong". I replied, "That's not good enough." (Teacher Journal, March 13, 2009)

I even found myself responding to students' answers differently. On a test over geometric shapes Govani asked me if his answer was correct. Instead of saying yes or no I asked him, "why do you think so?" After he provided an explanation I responded by saying that his reasoning sounded good. I affirmed his problem solving process rather than the accuracy of his solution. My words were supported by my actions; on every test it was written in all caps, "MUST SHOW ALL YOUR WORK in order to receive full credit". A student could have all correct answers but if no work was shown only partial credit was earned. This was crucial since answers alone did not provide an accurate representation of what a student knew and understood. For example on a test over geometry Raequan wrote that the measurement of one angle in a regular quadrilateral is 90º. This answer appears correct, but on the next problem a similar question was asked about a regular triangle. Her answer was 180º. Further probing lead me to find out that she had solved the quadrilateral problem by dividing 180 by 2 since two triangles where formed within the quadrilateral after drawing a diagonal. Her answer was the result of truth mixed with error that never would have been brought to light if I did not adhere to my standard that process is worth more than product.

Another change to my mathematics instruction was that I would ask more open-ended questions and allowed students to affirm, reject, or amend methods and solutions, rather than myself. On the test questionnaires one of the questions stated, ”How can you tell if your answer is wrong or right on your own?” The majority of the students responded with some form of double checking their work or noticing that the answer looks odd. Response of this type appeared on about 74 of the 92 questionnaires from chapters seven, nine, and eleven. Seventy-six out of the 92 included a written response that was NOT “I don’t know.” Of those 76, only two responded with a method that would require an action by me, “Ms. Grayer will mark it wrong” and “Ms. Grayer will walk over to you.” On April 23 one student questionnaire had the response, “Looking at the answer and comparing it to the question.” The student’s response shows that they were looking back at the problem to determine if their solution made sense.

In an interview session on April 7, SheeLen gave the following solutions to the addition problems with integers.

3 + 9 = 12 -3 + 9 = -12 -3 + (-9) = 12 3 + (-9) = -12

After being given the same problems in story format, he decided to change a few of his answers to:

3 + 9 = 12 -3 + 9 = 6 -3 + (-9) = -12 3 + (-9) = -6

He had mistakenly applied rules for determining if the answer should be positive or negative in multiplication and division problems to addition. However, once the problem was imbedded in context, SheeLen concluded that two of his original solutions did not fit the situation and was very comfortable changing them. SheeLen's behavior is an example of how students were becoming more flexible with their understanding, willing to modify their reasoning and solutions as new information was presented.

An interview with Boyd on April 7 gave a combination of open-ended questioning and student selected solutions. I asked Boyd if he could think of other problems besides 2 + (-5) whose solution was also -3 [using only addition or subtraction and the digits 2 and 5]. He came up with (-5) + 2, 2 – 5, and -5 – (-2). He reasoned that these where the only solutions because anything else would require going "to the left too much or too little or going to the right too much or too little from where you start at." With minimal restrictions, I had left Boyd open to come up with as many responses as he could and allowed him to justify why those were the only possible answers. In my daily notes on April 16 I recorded that:

> The warm up question I wrote for the students gave a fictitious student's solution to a problem to which they were asked to agree or disagree and give a supporting argument. Once students shared their reasoning with the class each student had to choose the reason that was the most sound and convincing to them. (Teacher Journal, April 16, 2009)

I left it open to students to accept or reject the solution to the given problem. The warm up was a prime example of how, through my action research, I began to see ways to deviate from assigning students problems to solve to giving students solutions and having them justify or reject the solutions based on their reasoning. Changing my questioning tactics led to a change in my instruction and how I interacted with students regarding the mathematics. Not only did I show an active belief in my philosophy that process is more important than product, but a belief that my students need to create and take ownership of the process that leads to the product began to manifest as well.

Conclusions

My research findings show that teacher-questioning habits have an influence on student actions and perceptions. At the beginning of the year I was very frustrated with the seemingly helplessness of my students. My students needed me to confirm every step they took while problem solving, every answer they got as a result, and the accuracy or relevancy of peer comments. I was not aware that my responses to their questions and actions enabled their helplessness. My yes/no responses essentially told students that I did not expect them to think for themselves and that I did not believe they were capable of accurately doing so. Good et al (1987) noticed this also in their study of student question asking behaviors. Their study results suggested that differential expectations lead to student passivity and that low teacher expectations resulted in low production from students. Once I began reintroducing “why?” into my instructional vocabulary and redirecting student questions towards their peers, I no longer became the sole source of authority and knowledge. Fuchs et al (1996) stated, “children do not naturally develop constructive interactional patterns without explicit instruction” (p. 635). To be explicit, I modeled the questioning of students explanations so that their peers could see what to say and know that questioning one another was acceptable. At one point during the research I had to very directly let students know that “I don’t know” or “I don’t get it” were not adequate verbalizations of confusion. The phrase “I don’t get it” does not provide sufficient information to know where communication or understanding broke down for the student giving the explanation. By the end of the research period students could give more specific vocalizations of their misunderstandings.

Fraivillig, Murphy, and Fuson (1999) support my conclusion that it is important for students to learn how to become better explainers of what does not make sense to them. These authors state that it is the student’s role to be engaged, explain and justify solution methods, make sense of peers’ methods, work collaboratively, and challenge peers. “Through critically examining others’ reasoning and participating in the resolution of disagreements, students learn to monitor their thinking in the service of reasoning about important mathematical concepts” (Pape, Bell, & Yetkin, 2003, p. 181). When students present their interpretations to the class so that peers and the teacher can question, contradict, or build upon them, a classroom that is focused on reasoning is created.

Implications

From embarking on this action research I now have “why” back in my vocabulary and I plan on giving it a permanent home. Beginning on Day One in my mathematics courses I will put forth the message through my words and actions that students will be expected to reason and push their peers to do so also. My research and literature show that students are able to engage in and initiate intellectual mathematics discussion about solutions and methods if given the tools and opportunities. My students became adequate at giving verbal explanations of their problem solving but struggled to do so in written form. One student stated with frustration while trying to complete her Friday journal that it is much easier to say what she means than to write it down. In the future I will need to incorporate more writing so that my students are effective in both modes of communication. I already have a tool in place, the students' math journals, which can be used to develop students' writing skills. Along with note taking the journals can be used for students to dictate explanations or problem-solving strategies presented that they understand and use.

It will also be beneficial to incorporate more sharing of explanations in pairs. Unfortunately, in a large group discussion some voices got lost or were never heard. I only need to look back on student responses on the questionnaires and in my journal to see who the more vocal students were. Having a student explain to one other person will help create a less intimidating atmosphere and increase engagement of all students. The one day I did have students partner up before responding to a Friday journal prompt gave me a glimpse into what could be if students shared in pairs. Students who were normally quiet were verbalizing their understandings and drawing unique examples to support their explanations. Continuing to implement new questioning habits into my instruction and allowing opportunities for students to explain and validate problem solving helps to create the kind of classroom that supports student construction of knowledge. It also will provide students with the ability to justifiably respond that their solutions are reasonable or not.

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Appendix A

<u>Teacher Journal Prompts:</u>

1. What significant or surprising questions were raised by students this week?
    a. How did I respond?
2. What significant or surprising explanations were given by students this week?
    a. How did I respond?
3. Who had the lead role in the explaining and/or questioning interactions for this week, the students or me? Explain.
4. What progress has the class or a particular student made towards being able to self-determine answer reasonability?
5. Who were the lead questioners? Explainers? Non-participants?
6. What changes can I make to bring the non-participants into the fold?
7. What did I learn this week that will guide my questioning or responses to students questions or explanations next week?
8. What were some tensions I felt this week between my role as teacher and researcher?

<u>Interview Questions:</u> for research question #1
What is the problem asking you?
What did you do to solve this problem?
Why did you choose to solve it that way?
Could you have solved it another way? How or show me?

<u>Interview Questions:</u> for research question #2
Is your answer reasonable?/Do your results answer the question? How do you know?
Are there any other possible answers? How do you know? Give me an example?

Appendix B

Student Questionnaire:

1. If you could only pick 1 to 3 peers from class to explain how they solved a problem who would it be and Why? Give a detailed reason for each person.

2. Who asks the best questions in class? Why?

3. Does Ms. Grayer asks students questions?

    a. What kinds of questions. Give an example.

4. Has Ms. Grayer asked you a question?
    a. What did she ask?

    b. Were you able to answer?

    i. Why? (Circle all that apply)
        1. Didn’t understand the question
        2. Didn’t know how to solve the problem
        3. Not enough time to figure it out
        4. Someone else blurted out the answer
        5. Wasn’t paying attention/didn’t know what the question was.
        6. Other ______________________________________
    ii. How did Ms. Grayer respond? (Circle all that apply)
        1. With encouragement
        2. With disappointment
        3. With assistance/help
        4. Moved on to someone else
        5. Waited for your answer
        6. Other ______________________________________

5. Who explanation makes how to solve a problem easier to understand, Ms. Grayer’s or a peer’s? Why?

6. When you’re confused, do you know what kinds of questions to ask to help you understand?

7. If a classmate was confused do you think that you could explain the problem or a way to solve it so that they understand? Why or why not?

Appendix C

<u>Student Questionnaire:</u> **Test Chapter: __________** **Date: _______**

1. If you could only pick 1 to 3 peers from class to explain how they solved a geometry problem (angles and lines) who would it be and why? (give a detailed reason for each person)

2. Who asks the best questions in class? and why?

3. How did Ms. Grayer respond when a student asks a question? (circle all that apply)
    1. With encouragement
    2. With disappointment
    3. Gives the answer
    4. Ask, "what do you think?"
    5. Ask another student to answer the question?
    6. Other ___________________________________

4. Whose explanation makes how to solve a problem easier to understand, Ms. Grayer's or a classmates? Why?

5. When you're confused, do you know what specific questions to ask to help you understand? (Not, I don't get it.)

6. If you knew the answer to a problem and a classmate was confused, do you think that you could explain the problem or a way to solve it so that they can figure it out too? Why or why not?

7. How can you tell if your answer is wrong or right on your own?

Appendix D

Student Questionnaire: **Test Chapter: __________** **Date: ___________**

1. If you could only pick 1 to 3 peers from class to explain how they solved a geometry problem (angles and lines) who would it be and why? (give a detailed reason for each person)

2. Who asks important questions in class? What makes a question “important”?

3. How does Ms. Grayer respond when a student asks a question? (circle all that apply) (*star the one she does the most)
    1. Gives the answer
    2. Ask, “what do you think?” or “is it?”
    3. Ask another student or the class to answer the question.
    4. Ignores or does not respond
    5. With excitement or intrigue (“great question!” or “hmm…let’s think about that”)
    6. Other ____

4. Who does more explaining of problems or how to solve a problem in class, Ms. Grayer or students?

5. Which is valued more in our math class, right answers or explaining/showing what to do to get the answer? How can you tell?

6. After solving a problem do you ask yourself if your answer makes sense? Why or why not?

7. How can you tell if your answer is wrong or right on your own?

Appendix E

<u>Student Questionnaire:</u> **Test Chapter: ___________** **Date: ____________**

1. If you could only pick 1 to 3 peers from class to explain how they solved a geometry problem (angles and lines) who would it be and why? (give a detailed reason for each person)

2. How does Ms. Grayer respond when a student asks a question? (circle all that apply) (*star the one she does the most)
   1. Gives the answer
   2. Ask, "what do you think?" or "is it?"
   3. Ask another student or the class to answer the question.
   4. Ignores or does not respond
   5. With excitement or intrigue ("great question!" or "hmm…let's think about that")
   6. Other ____

3. Who does more explaining of problems or how to solve a problem in class, Ms. Grayer or students?

4. Which is more important in our math class, right answers or explaining/showing what to do to get the answer? How can you tell?

5. After solving a problem do you ask yourself if your answer makes sense? Why or why not?

6. How can you tell if your answer is wrong or right on your own?

Appendix F

<u>Daily Student Questionnaire</u> **Lesson:___________** **Today's Date _________**

1. Did Ms. Grayer ask students questions today? YES or NO
   a. Give an example?

2. Did Ms. Grayer ask you a question today? YES or NO
   a. What did she ask?

   b. Were you able to answer? YES or NO
      i. If no, why? (circle all that apply)
         1. Didn't understand the question
         2. Didn't know how to solve the problem
         3. Not enough time to figure it out
         4. Someone else blurted out the answer
         5. Wasn't paying attention / didn't know what the question was.
         6. Other ___________________________________

      ii. If yes, how did Ms. Grayer respond to your answer? (circle all that apply)
         1. With encouragement
         2. With disappointment
         3. With assistance/help
         4. Moved on to someone else
         5. Waited for your answer
         6. Other ___________________________________

3. Who gave a really good explanation today? What made it "really good?

Appendix G

Daily Student Questionnaire **Lesson:_____________** **Today's Date ___________**

1. Did Ms. Grayer ask you a question today? YES or NO
   a. What did she ask?

   b. Were you able to answer? YES or NO

If NO, why? (Circle all that fit)

i. Didn't understand the question
ii. Didn't know how to solve the problem
iii. Not enough time to figure it out
iv. Someone else blurted out the answer
v. Wasn't paying attention/didn't know what the question was.
vi. Other
____

If YES, how did Ms. G respond? (Circle all that fit)

vii. With encouragement
viii. With disappointment
ix. With assistance/help
x. Moved on to someone else
xi. Waited for your answer
xii. Other
____

3. Did students have a chance to explain their problem solving today? YES or NO
Did you understand their explanation? Describe why or why not?

Appendix H

<u>Daily Student Questionnaire</u> **Lesson:___________** **Today's Date __________**

1. Did Ms. Grayer ask you a question today? YES or NO
   a. What did she ask?

   b. Were you able to answer? YES or NO

If **NO**, why? (Circle all that fit)

i. Didn't understand the question
ii. Didn't know how to solve the problem
iii. Not enough time to figure it out
iv. Someone else blurted out the answer
v. Wasn't paying attention/didn't know what the question was.
vi. Other ________________________________

If **YES**, how did Ms. G respond? (Circle all that fit)

vii. Said, "good answer" or "correct" or "yes!"
viii. Said, "no" or "almost" or "not really"
ix. Asked *another student* to build on your answer or add more.
x. Asked *you* to build on your answer or add more.
xi. Said, "show me" or "come explain up front"
xii. Other ________________________________

2. Did students ask each other to explain problem solving today in class? YES or NO

3. Did students have a chance to explain their problem solving today? YES or NO
   a. Restate the explanation a student gave.

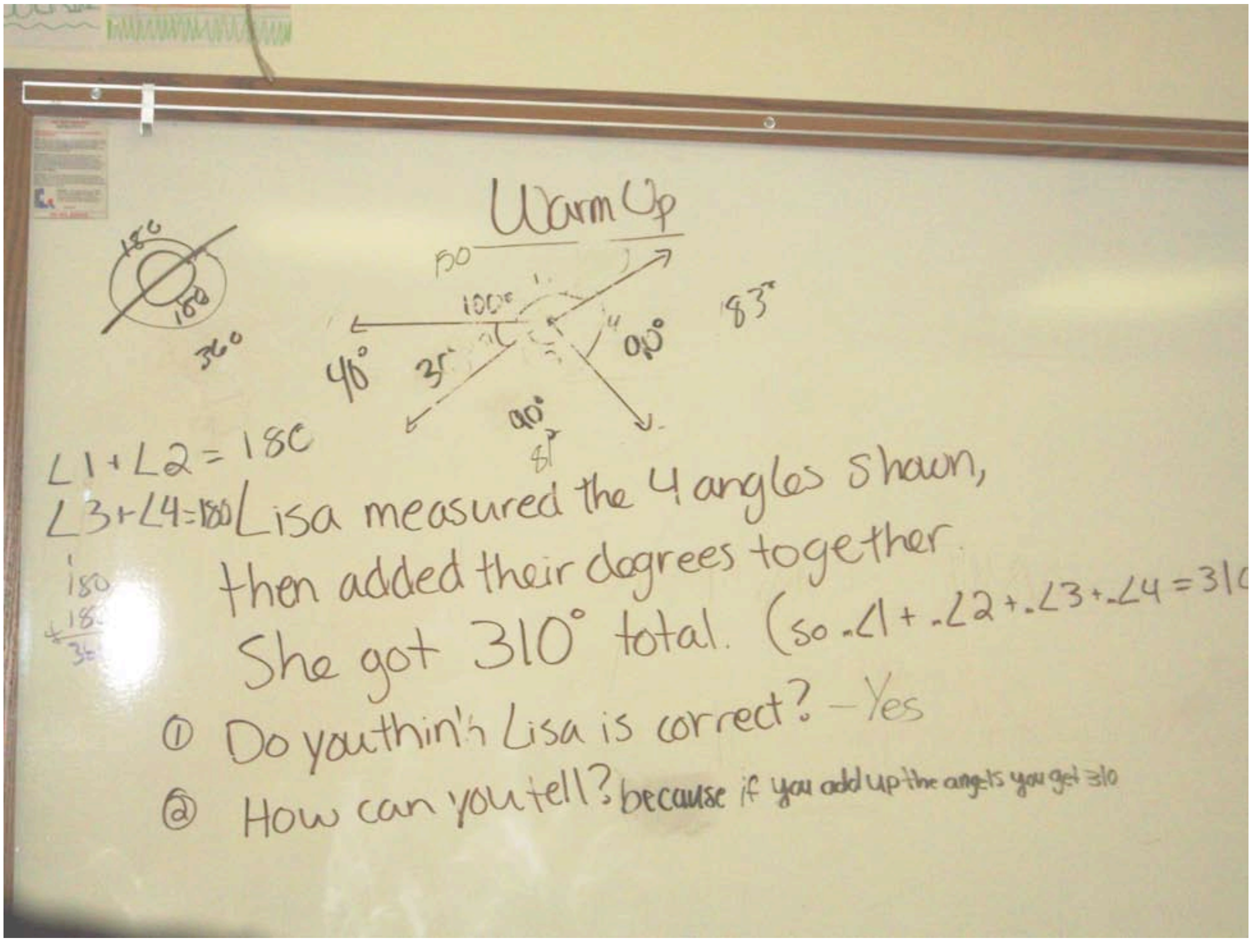
   b. Does it make sense? Why or Why not?

Appendix I

In-class Questioning:

Initials of student asking	S=Student asked T=Teacher asked	Question Type Code	S=asked to student T=asked to teacher	Additional Input	Code Key
					E=elaboration
					Y=why
					✓=what's wrong
					T=turn question back on asker
					?=I don't understand/get it/clarify
					R=repeat or paraphrase
					M=more responses

Appendix J



Warm Up
∠1 + ∠2 = 180
∠3 + ∠4 = 180
Lisa measured the 4 angles shown, then added their degrees together.
She got 310° total. (so ∠1 + ∠2 + ∠3 + ∠4 = 310
① Do you think Lisa is correct? – Yes
② How can you tell? because if you add up the angels you get 310